DC Electric Circuits: The RC Circuit

Goals and Introduction

We know from class and our other lab activities that when a current is run through a resistor, there is a potential difference observed across the resistor. As the current flows through the resistor, energy is dissipated in the form of light and heat, as you may recall from the lab activity "DC Electric Circuits: Resistance and Ohm's Law." In the simplest case, when a DC source is connected directly to the resistor, this potential difference is the same as that of the source.

A capacitor is a device that stores electric charge. When a potential difference is applied across a capacitor by connecting it to a power source, current will flow and a net electric charge of equal amount and opposite sign will build up on the two, separate, conducting regions within the capacitor. This continues until the potential difference across the capacitor, due to the charge stored there, is equal in magnitude to the potential difference of the source.

In any circuit, there is some small resistance, which affects the time it takes for the capacitor to become "fully charged." The rate at which this occurs is affected by the resistance, *R*, and the capacitance of the capacitor, *C*. A similar effect is observed when the capacitor is connected across a resistor, instead of the power source, and the capacitor discharges while driving an electric current in the circuit that decays over time. In modeling either the charging, or discharging, of the capacitor, we use as a value called the *time constant* for the circuit, given by Eq. 1 for an *RC* circuit. The unit of the time constant is seconds.

$$\tau = RC \qquad \text{(Eq. 1)}$$

The *RC* circuit is a circuit with a resistor and capacitor connected in series. If there are multiple resistors, we must express the equivalent resistance that can be modeled as being in series with the capacitor. When we look at the discharging of a capacitor, the potential difference across the capacitor will decay over time, as described by Eq. 2.

$$\Delta V_C = \Delta V_{C,\max}\ e^{-t/\tau} \qquad \text{(Eq. 1)}$$

Here, we see the time constant appearing in the exponential function. If we examine the value of the potential difference when one time constant has passed (when $t = \tau$), since beginning to discharge the capacitor, we can see that the potential difference will be 36.8% of its initial, maximum value.

$$\Delta V_C = \Delta V_{C,\max} \; e^{-\tau/\tau} = \Delta V_{C,\max} \; e^{-1}$$

$$\Delta V_C = \Delta V_{C,\max} \quad 0.368$$

We could also examine what the potential difference should be, in terms of the initial, maximum value, when we allow two times constants, or more to pass. Here, we would just have to set *t* to equal any whole number multiple of the time constant to calculate the percentage of the initial potential difference that should still exist at that time.

In today's lab, you will discharge a capacitor through different resistor combinations and monitor the potential difference across the capacitor. By recording the potential difference at several times, you will be able to plot the potential difference as a function of time and determine the time constant by fitting an exponential curve to the graph and examining the model equation of the curve fit. This can be compared to the predicted value based on the known equivalent resistance that is in series with the capacitor and the capacitance.

In order to perform the experiment, we will need to first use the discharge model to determine the resistance of the voltmeter (digital multimeter in this lab). When acting as a voltmeter, the device has a very high resistance. This is, by design, to severely limit the current that would flow through the device, allowing it to function as a tool for measuring the potential difference across different points in a circuit without drastically affecting the circuit, or measurement. Though the resistance is quite large, we will still find that when connected in series with the charged capacitor alone, a current will flow and the potential difference will decrease. In this lab, you should treat the voltmeter as a resistor, as well as a tool for monitoring the potential difference across the capacitor.

Goals: (1) Perform measurements to model the discharging of a capacitor
(2) Determine the resistance of the digital multimeter on a particular setting, when it is behaving as a voltmeter.
(3) Test the discharge model by including other resistors in the circuit and determining the equivalent resistance of the circuit.

Procedure

Equipment – electric connection board, double-pole double-throw switch, 8 wires, 0 – 30 V DC 1 A wall power source, 2 resistors of different resistance, a capacitor, digital multimeter, stopwatch, two alligator clips

NOTE: The double-pole double-throw switch is a tool for rapidly changing the configuration of the circuit. In this experiment, when the switch is open (vertical) the capacitor will be in series with the resistance of the voltmeter alone. When the switch is closed forward (away from the student), the resistance of the voltmeter will be in parallel with the resistor at the top of the connection board. The equivalent resistance of that combination is in series with the capacitor. Lastly, when the switch is closed backward (towards the student), the resistance of the voltmeter will be in parallel with the resistor at the bottom of the connection board. The equivalent resistance of that combination is in series with the capacitor.

1) Plug one wire into the “COM” port on one of the multimeters and another into the “V- - -” port. Attach the alligator clips to the free ends of the wires plugged into the multimeter, and clip them across the two ends of one of the resistors. Turn the knob on the multimeter to the area marked with Ω and set the dial to the best possible setting for precision. Note that when you are on a setting with a “K”, the meter is reading in thousands of Ohms. If the meter shows a one with a line next to it, this means the resistance is larger than the current setting. If this is the case, turn the dial one click counter-clockwise until you can get a reading of the resistance. **Record** the resistance of the resistor, being sure to convert your measurement to ohms.

2) Unclip the first resistor and clip the meter to the other resistor. Turn the dial on the meter to the best possible setting for precision, and **record** the resistance of the second resistor, being sure to convert your measurement to ohms. Make note of which resistor is which (one will have a greater resistance than the other)! Then, remove the alligator clips.

3) Examine the capacitor and **record** the capacitance that you find printed there. Note that if you see a unit of “MF”, or “mF” on the capacitor, it very likely stands for “microfarads”, not “millifarads.” Why would someone manufacture these things in this way? ... And now, on with the show! ☺

4) Connect the resistor with the lower resistance to two adjacent posts at the top of the connection board. Then, connect the resistor with the greater resistance to two adjacent posts at the bottom of the board.

5) Attach an alligator clip to each lead wire from the capacitor. Then, connect one of the posts on the bottom resistor to one of the alligator clips on the capacitor, by plugging in to the back of the clip. After that, connect one of the posts on the top resistor to THE SAME alligator clip on the capacitor, by plugging into the back of the wire already there.

6) Set the double-pole double-throw switch next to the connection board. Connect the bottom post on one side of the switch to the post on the bottom resistor that currently has nothing

plugged into it. Connect the middle post on the same side of the switch to the end of the capacitor with nothing connected currently, by plugging in to the back of the alligator clip there.

7) Check that the 0 – 30 V DC 1A wall power supply is turned off and that the knob is turned completely counter-clockwise. Connect the top post on the same side of the switch to the red port on the 0 – 30 V DC 1A wall power supply. Connect the post on the top resistor, with nothing else connected currently, to the black port on the wall power supply.

8) Plug one wire into the “COM” port on the multimeter and another into the “V- - -” port. Turn the dial on the meter so that it points to “20” in the section labeled as “V- - -” on the outer edge of the meter. Then, connect the “COM” wire to the capacitor lead where the other resistors are connected, and connect the other lead from the meter to the other capacitor lead. This meter will act as the voltmeter during the experiment, and measures the potential difference across the capacitor at any moment. Your completed setup should look similar to the Figure 1 below. If you are unsure, check with your TA!

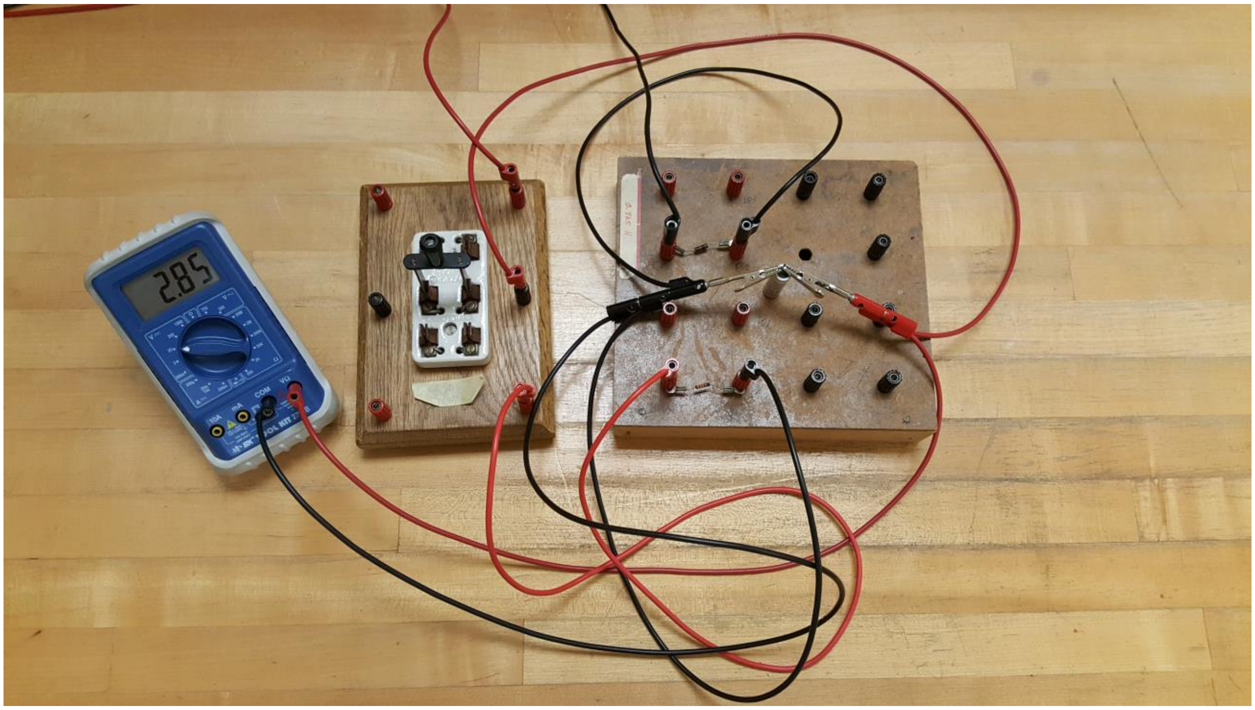

Figure 1

9) Move the switch forward so that it is closed in the forward position. Then, turn on the wall power supply and slowly turn the knob until the reading on the voltmeter is about 15 V. **Record** the potential difference. This will be the maximum value of the potential difference on the capacitor and the value of the potential difference when *t* = 0 in the next step.

10) For this next step, you and your partner should agree upon a set of times to record the potential difference across the capacitor. Recording at least every 2 or 3 seconds is probably best. Whatever you choose, the goal is to be able to create a table of times, beginning from $t = 0$, and corresponding values of the potential difference across the capacitor. Have one lab partner ready with the stopwatch and the other with a pen and paper. The one partner should lift the switch so that it remains open in the vertical position, while the other partner starts the stopwatch. **Record** the potential difference on the voltmeter **and** the time on the stopwatch until you feel you cannot reliably measure the potential difference, or until you have 30 recorded values (whichever happens first).

NOTE: You should not turn the dial on the multimeter, as the resistance may change.

11) Close the switch forward so that the capacitor charges again. **Record** the potential difference on the capacitor and repeat the previous step again (step 10).

12) Close the switch forward so that the capacitor charges again. **Record** the potential difference on the capacitor. This time, rather than leaving the switch in the vertical position, we will flip it and close the switch into the bottom position. In this configuration, the capacitor will be discharging through the resistor in the two bottom posts and the resistance of the voltmeter in parallel. Later, we will calculate the equivalent resistance based on your other analyses. Have one lab partner ready with the stopwatch and the other with a pen and paper. The one partner should quickly flip the switch so that it is closed in the bottom position, while the other partner starts the stopwatch. **Record** the potential difference on the voltmeter **and** the time on the stopwatch until you feel you cannot reliably measure the potential difference, or until you have 30 recorded values (whichever happens first).

13) Repeat step 12.

14) Now, close the switch forward so that the capacitor charges again. **Record** the potential difference on the capacitor. This time, rather than moving the switch, we will just turn off the power supply by flipping the switch on the wall. In this configuration, the capacitor will be discharging through the resistor in the two top posts and the resistance of the voltmeter in parallel. Later, we will calculate the equivalent resistance based on your other analyses. Have one lab partner ready with the stopwatch and the other with a pen and paper. The one partner should turn off the wall power supply by flipping the switch, while the other partner starts the stopwatch. **Record** the potential difference on the voltmeter **and** the time on the stopwatch until you feel you cannot reliably measure the potential difference, or until you have 30 recorded values (whichever happens first).

15) Repeat step 14. When you are done, turn the knob on the wall power supply all the way counter-clockwise.

As always, be sure to organize your data records for presentation in your lab report, using tables and labels where appropriate.

Data Analysis

Using your data from step 10, create a scatter plot of potential difference across the capacitor (y – axis) versus time (x – axis). Then add a trendline to the plot, selecting to model the data with an "exponential" line. Be sure to choose to "Display the Equation" on the chart. Also, remember to label axes and title your plot accordingly.

Repeat the previous plot, using the data from step 11 instead.

Compare Eq. 2 to the equation displayed on each plot, and determine the time constant, τ , for each plot by examining the exponent of the function.

Use the capacitance and the time constant in each case to calculate the resistance of the voltmeter in each case.

Compute the mean value of the resistance of the voltmeter and use this value for the its resistance in future calculations.

Using your data from step 12, create a scatter plot of potential difference across the capacitor (y – axis) versus time (x – axis). Then add a trendline to the plot, selecting to model the data with an "exponential" line. Be sure to choose to "Display the Equation" on the chart. Also, remember to label axes and title your plot accordingly.

Repeat the previous plot, using the data from step 13 instead.

Compare Eq. 2 to the equation displayed on each plot, and determine the time constant, τ , for each plot by examining the exponent of the function.

Use the capacitance and the time constant in each case to calculate the equivalent resistance.

Compute the mean value of the equivalent resistance and note this as the experimental value for the equivalent resistance of the voltmeter and bottom resistor in parallel.

Using the resistance you measured for the bottom resistor at the beginning of the lab activity and the resistance of the voltmeter that you found, calculate a predicted value for the equivalent resistance for their parallel combination.

Using your data from step 14, create a scatter plot of potential difference across the capacitor (y – axis) versus time (x – axis). Then add a trendline to the plot, selecting to model the data with an "exponential" line. Be sure to choose to "Display the Equation" on the chart. Also, remember to label axes and title your plot accordingly.

Repeat the previous plot, using the data from step 15 instead.

Compare Eq. 2 to the equation displayed on each plot, and determine the time constant, τ, for each plot by examining the exponent of the function.

Use the capacitance and the time constant in each case to calculate the equivalent resistance.

Compute the mean value of the equivalent resistance and note this as the experimental value for the equivalent resistance of the voltmeter and top resistor in parallel.

Using the resistance you measured for the top resistor at the beginning of the lab activity and the resistance of the voltmeter that you found, calculate a predicted value for the equivalent resistance for their parallel combination.

Question 1: How could the voltmeter and the capacitor be used to measure the resistance of an unknown resistor? Describe a process that would allow you to determine the value of an unknown resistance using a charged capacitor, a voltmeter, a stopwatch, and some wires.

Question 2: Examine your plots of potential difference versus time and consider Eq. 2, along with the equations displayed on the chart. Did we confirm the exponential discharge model? Explain and support your conclusion. Consider what percentage the potential difference had fallen to after one time constant.

Error Analysis

Calculate the percent difference between the two values you found for the resistance of the voltmeter.

$$\%diff = \frac{|R_1 - R_2|}{R_1 + R_2 \;/2} \times 100\%$$

Calculate the percent error between the experimental value and predicted value of the equivalent resistance for the voltmeter in parallel with the bottom resistor.

$$\%error = \frac{\left|R_{\text{predict}} - R_{\text{exp}}\right|}{R_{\text{predict}}} \times 100\%$$

Calculate the percent error between the experimental value and predicted value of the equivalent resistance for the voltmeter in parallel with the top resistor.

Question 3: Were your results consistent? What sources of error might exist that affected the results of the experiment, and what could be done to try to overcome those difficulties in order to get a more accurate measurement?

Questions and Conclusions

Be sure to address Questions 1 through 3 and describe what has been verified and tested by this experiment. What are the likely sources of error? Where might the physics principles investigated in this lab manifest in everyday life, or in a job setting?

Pre-Lab Questions

Please read through all the instructions for this experiment to acquaint yourself with the experimental setup and procedures, and develop any questions you may want to discuss with your lab partner or TA before you begin. Then answer the following questions and type your answers into the Canvas quiz tool for "DC Electric Circuits: Resistors in Combination," and submit it before the start of your lab section on the day this experiment is to be run.

PL-1) Christopher examines his plot of potential difference versus time and notices that the exponent reads "-2*t*." He compares this to Eq. 2 in the lab document to determine the time constant for his experiment. What is the time constant? Express your answer in seconds, s.

PL-2) Given the set-up of the circuit described in the procedure, when the double-pole double-throw switch is in the vertical position, the capacitor will

A) discharge through the bottom resistor only.

B) remain charged until you flip the switch to the other side.

C) discharge through the top resistor only.

D) discharge through the resistance of the voltmeter.

PL-3) Emily finds that the resistance of the voltmeter is 6.00×10^6 Ω. When it is connected in parallel with a 10,000 Ω resistor, the equivalent resistance is (answer in units of ohms, Ω)

PL-4) Christopher and Emily examine an *RC* circuit with a time constant of 0.35 s. If the initial potential difference was 12 V, what was the potential difference across the capacitor after two time constants had passed, while discharging. Express your answer in volts, V.

PL-5) After recording many measurements, Emily notices that the precision of the measurement is becoming less because of the setting on the meter. She should

A) continue to record values until she has thirty values, or it becomes impossible to distinguish one reading from the next.

B) turn the dial on the meter for added precision.

C) assume she has enough measurements and proceed to the next step.

D) recharge the capacitor and start over on a different setting.